

LONG-TERM DYNAMICS  
OF SMALL BODIES IN THE SOLAR SYSTEM

GRANT NAG5-7761

FINAL REPORT

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Principal Investigator

Dr. Matthew J. Holman

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Smithsonian Institution  
Astrophysical Observatory  
Cambridge, Massachusetts 02138

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John Grant, 077.0, NASA Headquarters  
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## **Final Report for NASA Planetary Geology and Geophysics Grant NAG5-7761**

**PI: Matthew J. Holman**

**Period of Performance: January 1999 – December 2001**

As part of the NASA Planetary Geology and Geophysics program, Prof. Norm Murray (CITA) and I have been conducting investigations of the long-term dynamics of small bodies in the outer solar system. This grant, and its successor NAG5-10365, supports travel for collaboration by the Investigators and also supports Murray during an annual one month visit to the CfA for further collaboration. In the course of this grant we made a number of advances in solar system dynamics. For example, we developed an analytic model for the origin and consequence of chaos associated with three-body resonances in the asteroid belt. This has been shown to be important for the delivery of near Earth objects. We later extended this model to three-body resonances among planets. We were able to show that the numerically identified chaos among the outer planets results from a three-body resonance involving Jupiter, Saturn, and Uranus. The resulting paper was awarded the 1999 Newcomb Cleveland award from the AAAS. This award singles out one paper published in Science each year for distinction.

### **Publications related to this grant:**

#### **1. The role of chaotic resonances in the Solar System**

Murray, N.; Holman, M.  
Nature, 773-779 (2001).

#### **Abstract**

Our understanding of the Solar System has been revolutionized over the past decade by the finding that the orbits of the planets are inherently chaotic. In extreme cases, chaotic motions can change the relative positions of the planets around stars, and even eject a planet from a system. Moreover, the spin axis of a planet-Earth's spin axis regulates our seasons-may evolve chaotically, with adverse effects on the climates of otherwise biologically interesting planets. Some of the recently discovered extrasolar planetary systems contain multiple planets, and it is likely that some of these are chaotic as well.

#### **2. Chaos in the Solar System**

Lecar, Myron; Franklin, Fred A.; Holman, Matthew J.; Murray, Norman J.  
Annual Review of Astronomy and Astrophysics, 39, 581-631 (2001).

#### **Abstract**

The physical basis of chaos in the solar system is now better understood: In all cases investigated so far, chaotic orbits result from overlapping resonances. Perhaps the clearest examples are found in the asteroid belt. Overlapping resonances account for its Kirkwood gaps and were used to predict and find evidence for very narrow gaps in the outer belt. Further afield, about one new "short-period" comet is discovered each year. They are believed to come from the "Kuiper Belt" (at 40 AU or more) via chaotic orbits produced by mean-motion and secular resonances with Neptune. Finally, the planetary system itself is not immune from chaos. In the inner solar system, overlapping secular resonances have been identified as the possible source of chaos. For example, Mercury, in 1012 years, may suffer a close encounter with Venus or plunge into the Sun. In the outer solar system, three-body resonances have been identified as a source of chaos, but on an even longer time scale of 109 times the age of the solar system. On the human time scale, the planets do follow their orbits in a stately procession, and we can predict their trajectories for hundreds of thousands of years. That is because the mavericks, with shorter instability times, have long since been ejected. The solar system is not stable; it is just old!

#### **3. Production of Star-grazing and Star-impacting Planetesimals via Orbital Migration of Extrasolar Planets**

Quillen, A. C.; Holman, M.  
The Astronomical Journal, 119, 397-402 (2000).

### Abstract

During orbital migration of a giant extrasolar planet via ejection of planetesimals (as studied by Murray et al. in 1998), inner mean-motion resonances can be strong enough to cause planetesimals to graze or impact the star. We integrate numerically the motions of particles which pass through the 3:1 or 4:1 mean-motion resonances of a migrating Jupiter-mass planet. We find that many particles can be trapped in the 3:1 or 4:1 resonances and pumped to high enough eccentricities that they impact the star. This implies that for a planet migrating a substantial fraction of its semimajor axis, a fraction of its mass in planetesimals could impact the star. This process may be capable of enriching the metallicity of the star at a time when the star is no longer fully convective. Upon close approaches to the star, the surfaces of these planetesimals will be sublimated. Orbital migration should cause continuing production of evaporating bodies, suggesting that this process should be detectable with searches for transient absorption lines in young stars. The remainder of the particles will not impact the star but can be ejected subsequently by the planet as it migrates further inward. This allows the planet to migrate a substantial fraction of its initial semimajor axis by ejecting planetesimals.

#### 4. The Origin of Chaos in the Outer Solar System

Murray, N.; Holman, M.  
Science, 283, 1877 (1999).

#### 5. Dynamical Chaos in the Wisdom-Holman Integrator: Origins and Solutions

Rauch, Kevin P.; Holman, Matthew  
The Astronomical Journal, 117, 1087-1102 (1999).

### Abstract

We examine the nonlinear stability of the Wisdom-Holman (WH) symplectic mapping applied to the integration of perturbed, highly eccentric ( $e \sim 0.9$ ) two-body orbits. We find that the method is unstable and introduces artificial chaos into the computed trajectories for this class of problems, unless the step size chosen is small enough that periape is always resolved, in which case the method is generically stable. This "radial orbit instability" persists even for weakly perturbed systems. Using the Stark problem as a fiducial test case, we investigate the dynamical origin of this instability and argue that the numerical chaos results from the overlap of step-size resonances; interestingly, for the Stark problem many of these resonances appear to be absolutely stable. We similarly examine the robustness of several alternative integration methods: a time-regularized version of the WH mapping suggested by Mikkola; the potential-splitting (PS) method of Duncan, Levison, Lee; and two original methods incorporating approximations based on Stark motion instead of Keplerian motion (compare Newman et al.). The two fixed point problem and a related, more general problem are used to conduct a comparative test of the various methods for several types of motion. Among the algorithms tested, the time-transformed WH mapping is clearly the most efficient and stable method of integrating eccentric, nearly Keplerian orbits in the absence of close encounters. For test particles subject to both high eccentricities and very close encounters, we find an enhanced version of the PS method-incorporating time regularization, force-center switching, and an improved kernel function-to be both economical and highly versatile. We conclude that Stark-based methods are of marginal utility in N-body type integrations. Additional implications for the symplectic

#### 6. Long-Term Stability of Planets in Binary Systems

Holman, Matthew J.; Wiegert, Paul A.  
The Astronomical Journal, 117, 621-628 (1999).

### Abstract

A simple question of celestial mechanics is investigated: in what regions of phase space near a binary system can planets persist for long times? The planets are taken to be test particles moving in the field of an eccentric binary system. A range of values of the binary eccentricity and mass ratio is studied, and both the

case of planets orbiting close to one of the stars, and that of planets outside the binary orbiting the systems center of mass, are examined. From the results, empirical expressions are developed for both (1) the largest orbit around each of the stars and (2) the smallest orbit around the binary system as a whole, in which test particles survive the length of the integration ( $10^4$  binary periods). The empirical expressions developed, which are roughly linear in both the mass ratio  $\mu$  and the binary eccentricity  $e$ , are determined for the range  $0.0 \leq e \leq 0.7-0.8$  and  $0.1 \leq \mu \leq 0.9$  in both regions and can be used to guide searches for planets in binary systems. After considering the case of a single low-mass planet in binary systems, the stability of a mutually interacting system of planets orbiting one star of a binary system is examined, though in less detail.

7. On the Origin of Chaos in the Asteroid Belt  
Murray, N.; Holman, M.; Potter, M.  
The Astronomical Journal, 116, 2583-2589 (1998).

#### Abstract

We consider the effect of gravitational perturbations from Jupiter on the dynamics of asteroids, when Jupiter is itself perturbed by Saturn. The presence of Saturn introduces a number of additional frequencies into Jupiter's orbit. These frequencies in turn produce chaos in narrow regions on either side of the chaotic zones associated with the mean motion resonances between the asteroids and Jupiter. The resonant arguments of these three-body resonances contain the longitudes of Jupiter and the asteroid together with either the secular frequency  $g_6$ , or the longitude of Saturn. Resonances involving the longitude of Saturn are analogs of the Laplace resonance in the Jovian satellite system. We show that many three-body resonances involving the longitude of Saturn are chaotic. We give simple expressions for the width of the chaotic region and the associated Lyapunov time. In some cases the chaos can produce a diffusive growth in the eccentricity of the asteroid that leads to ejection of the asteroid on times shorter than the age of the solar system. We give simple estimates for the diffusion time. Finally, we present the results of numerical integrations testing the theory.

